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THE VARIATION OF THE SALT CONCENTRATION AT THE DISCHARGE OF A RIVER INTO A SALINE WATER

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Abstract

A plume model is used to describe the variation of the salt concentration at the discharge of a river into a saline water. The integral model of the plume behavior consists of a set of ordinary differential equations derived from conservation of mass, momentum and salt concentration. The temperatures of the plume and ambient saline water are considered equal. The concentration of the salt in the river water is null. The saline water is assumed motionless. After release from the river, the concentration of the salt in the plume increases by mixing with the ambient saline water. The rate of mixing depends upon the local plume and ambient fluid properties such as velocity and salt concentration.

1 Introduction

River inflows and their associated buoyant plumes are a major source of nutrients, sediments and contaminants to coastal waters. Also, they are a major threat to coastal and marine ecosystems such as coral reefs and seagrass beds. The plume, which is characterized by low-salinity water from the river, can determine the water density structure and circulation systems for these regions. For these reasons, considerable effort has been made to monitor the circulation within river plumes in order to determine the transport and dispersal of river-borne matter in the coastal zone (see for example, [7], [11], [18], [24] and

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[25]). Sea surface salinity in coastal oceans is a direct indicator of riverine plumes and provides essential information about the ocean environment and ecosystem. For the freshwater plumes produced by river discharges buoyancy often leads to expansion primarily along the surface. The freshwater impact on salinity is mostly noted through density currents in the sea surface. River plumes exhibit a great variety of spatial scales and shapes. Based on the values of the Kelvin number, R_K, Garvine [12] sizes the river plumes into: (1) small-scale plumes and (2) large-scale plumes. The Kelvin number, R_K, is defined as,

$$R_K = \frac{f w_0}{\sqrt{b_0 h_0}} \tag{1}$$

where f is the local Coriolis parameter (frequency), w_0 the river width, h_0 the upper layer thickness, $b_0 = g(\rho_a - \rho_p)/\rho_a$, ρ_a the ambient density, ρ_p the buoyant fluid density and g the gravity acceleration. For small scale plumes ($R_K < 1$) the inertial effects are more important than earths rotation. For large scale plumes ($R_K \geq 1$) the effect of earths rotation becomes important. From another point of view, the river plumes are grouped as, [4]: (i) surface trapped or (ii) bottom attached. For the surface trapped plumes the fresh water forms a shallow, surface trapped layer that spreads over the ambient shelf water and exhibits strong vertical stratification. For the bottom attached plumes a significant portion of the plume spams the water column from surface to bottom.

There is not a single, unitary class of mathematical models that describe the river plumes behavior. Chatanantavet and Lamb [5] and Poggioli [22] developed a 1-D mathematical model based on the hydraulics of the open channel. Considering the ocean barotropic, inviscid, uniformly rotating, the 2-D inviscid barotropic vorticity equation was used in [13], [16], [17], [19], [25], and [26]. The semi-spectral 3-D primitive equation model proposed by Haidvogel et al. [14] was used and improved in [3], [9], [10], [21], [27]. Osadchiev and Zavialov [20] proposed a Lagrange mathematical model based on tracking and dissipation of imaginary individual particles of the river water discharged into the sea. Laboratory experiments used to simulate the plume behavior can be viewed in [15], [22].

The aim of the present paper is to describe the discharge of a river into a saline water using an integral plume model. The integral plume model was used to simulate the deep-sea gravity currents [1], melting of the glacial system [2], hydrothermal vents from fracture area at the sea floor, [6] and deep waters oil blowout plumes, [8]. This paper is organized as follows. Section 2 describes the integral plume model used in this work. The numerical solving of the present mathematical model is briefly presented in Sect. 3. The numerical experiments made and the results obtained are presented in Sect. 4. Finally, some concluding remarks are briefly mentioned in Sect. 5.

2 Model equations

Generally, it is assumed that marine plumes exhibit similar behaviour to aerial plumes. However, marine plumes show similar characteristics to aerial plumes only qualitatively. Quantitatively, they are different. The evolution of marine plumes is influenced significantly by a large number of factors, such as the bathymetry, the Coriolis force, the river discharge, winds, tides and other coastal processes. The case analysed in the present work describes the discharge of a river into a salt lake. Thus, from the beginning one can consider that the present plume is a surface trapped, small scale one. The temperatures of the river water and the saline water are the same. The effects of the winds and saline water internal currents are neglected. Tides do not occur in the present physical environment. The saline water is considered motionless. For a latitude of approximately 45°, the Coriolis frequency is equal to $f = 10^{-4}s^{-1}$. The reduced gravitational acceleration b_0 is on the order of, $b_0 \approx 0.1ms^{-2}$. The values of h_0 and w_0 used in this work are: $h_0 = 0.5m \div 1m$ and $w_0 = 50m$. Under these conditions, the values of the Kelvin number are: $R_K < 0.022$. For these Kelvin number values the effect of earth rotation is negligible.

The plume moves in the y direction (figure 1). Under the scenario previously outlined, the following balance equations were obtained by averaging over a cross-section of the plume with area $A = wh$ (w and h are respectively the width and height of the plume - see figures 1 and 2):

- Mass conservation

$$\frac{d}{dy}\left(\rho_p \ w \ h \ U\right) = \rho_a(w+2h)V_e \tag{2}$$

- Momentum conservation

$$\frac{d}{dy}\left(\rho_p \ w \ h \ U^2\right) = -C_D\rho_p U^2\rho_a(w+2h) \tag{3}$$

- Salt conservation

$$\frac{d}{dy}\left(\rho_p \ w \ h \ US_p\right) = \rho_a V_e(w+2h)S_a \tag{4}$$

where C_D is the drag coefficient, S is the concentration of the salt, U is the average plume velocity, V_e is the entrainment (mixing) velocity of the saline

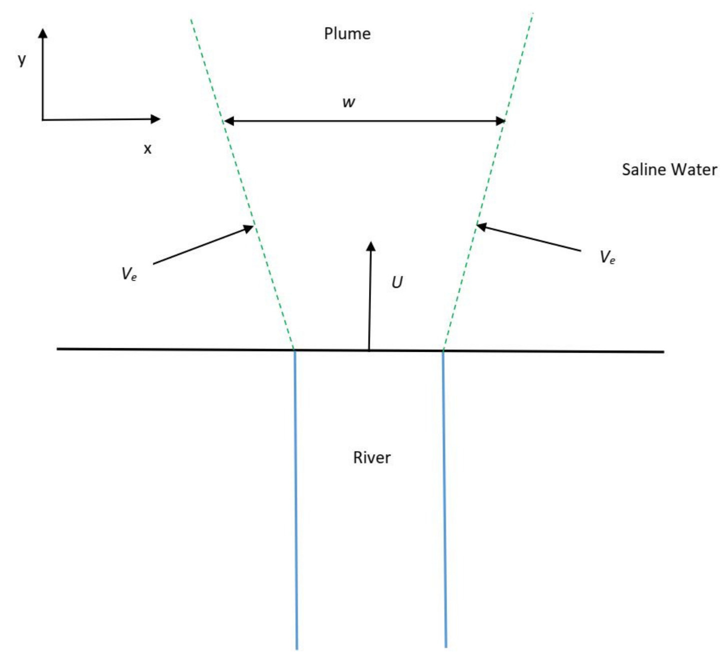

Figure 1: Plume moves in the y direction.

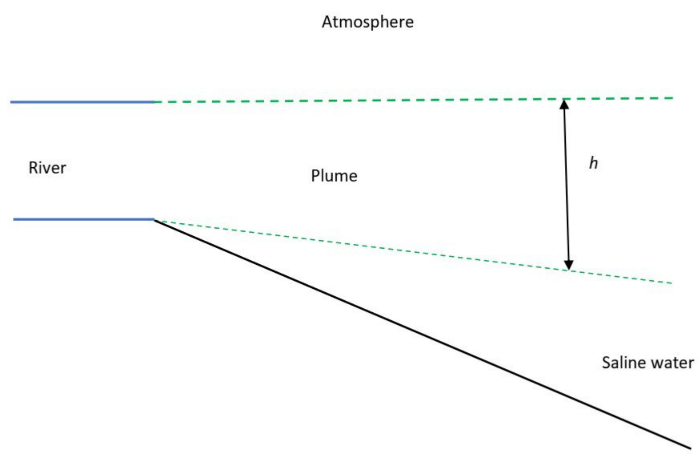

Figure 2: Plume area.

water into the plume, and ρ is the density. The subscripts a and p refer to the ambient saline water and plume, respectively. The entrainment surface into the plume is $(w + 2h)$. The entrainment velocity V_e was calculated with the well-known semi-empirical relation, [1],[6]:

$$V_e = \alpha U. \tag{5}$$

We have also assumed the followings:

- Constant width over thickness ratio,[1],

$$\frac{w}{h} = k = constant >> 1 \tag{6}$$

- The density of the plume and saline water are constant and approximately equal,

$$\rho_a \approx \rho_p \tag{7}$$

For the scenario previously mentioned, after some algebraic manipulations, the mathematical model reads as:

$$\frac{dw}{dy} = \frac{k\left(1 + \frac{2}{k}\right)}{2U}(2V_e + C_D U) \tag{8}$$

$$\frac{dU}{dy} = -\frac{k\left(1+\frac{2}{k}\right)}{w}(C_D U + V_e) \tag{9}$$

$$\frac{dS_p}{dy} = \frac{k\left(1+\frac{2}{k}\right)}{wU} V_e (S_a - S_p) \tag{10}$$

with the initial conditions :

$$y = 0, w = w_0, U = U_0, Sp = 0. \tag{11}$$

3 Method of solution

The mathematical model equations (4 -6) with the initial conditions (7) were solved numerically. The MATLAB routine ode45 was used (the present problem is not a stiff system of ordinary differential equations). The error tolerance considered was 10^{-6}. Different values were considered for the final y values such that the integration was stopped when,

$$U \leq 0.005 m/sec. \tag{12}$$

4 Results

The parameters of the present mathematical model are: C_D, S_a, a and k. According to [1], the value recommended for C_D is: $C_D = 0.013$. Usually, the value of a is determined experimentally. The value of this parameter remains a matter for debate. However, commonly accepted values vary from $a = 0.054$ for jets to $a = 0.083$ for plumes, [6]. These values represent in fact a parameterization of the turbulence. In this work the value used for a is: $a = 0.083$. In all numerical experiments a single value was assumed for S_a, i.e. $S_a = 1.0$. The geometric parameter k takes values in the range, $k = 50 \ldots 100$. The numerical simulations were made for the following values of U_0 and w_0:

$$U_0 = 0.1, 0.5, 1, 2, 5, 10 m/sec, w_0 = 50m.$$

Replacing in equation (8) the expression for the entrainment velocity V_e from (5), e.g. $V_e = \alpha U$, one obtains, after some elementary algebraic manipulations,

$$\frac{dw}{dy} = k\left(1+\frac{2}{k}\right)\left(\alpha + \frac{C_D}{2}\right) \tag{13}$$

Thus, the average plume velocity does not influence the width of the plume. For this reason, in figure 3, only a single curve is plotted. Figure 3 shows, according to the previous relation, that the variation of w versus y is linear and w increases with the increase in y. Applying the same algebraic manipulations, equation (10) reads as:

$$\frac{dS_p}{dy} = \frac{k\left(1+\frac{2}{k}\right)}{w}\alpha(S_\alpha - S_p). \tag{14}$$

The previous relation shows that the salinity of the plume S_p does not depend on the initial value of the plume average velocity. The increase in the plume average velocity increases the entrainment velocity and implicitly the turbulent mixing. Figure 4 shows that the variation of S_p versus y is different compared to the variation of the plume width w versus y. The salinity of the plume increases faster with the increase in y and reaches the value of ambient salinity for low values of y. The concentration gradient between the plume and the saline water decreases significantly with the increase in w. For values of w greater than 500 it becomes smaller than 0.01. For this reason, in figure 4, the maximum value of y is smaller than the value considered in figures 3 and 5. The average plume velocity depends on the discharge rate. The increase in U_0 increases the values of U (see figure 5). In figure 5 the maximum value selected for y is $y = 15000m$ (the same value as in figure 3). We made this selection to avoid the situation of three figures with three different abscissae. Figure 5 shows that the plume length increases with the increase in U_0 but, as previously mentioned, its composition remains practically the same. In all the test cases the value of k for a constant width over thickness radio is taken to be $k = 50$.

5 Conclusions

The variation of the salt concentration at the discharge of a river into a saline water was analysed using an integral the plume model. The temperatures of the plume and ambient saline water are considered equal. The concentration of the salt in the river water is null. The saline water is assumed motionless. The main issue investigated in the present paper is the influence of the river discharge rate on the plume composition and dimensions. The results provided by the present model and presented in the previous section show that the plume salinity and width do not depend on the discharge rate. The discharge rate influences only the plume length.

References

[1] G. Alendal, H. Drange, P.M. Haugan,*Modelling of deep-sea gravity currents using an integrated plume model*, Geophysical Monograph, 85 (1994), 237 – 246.

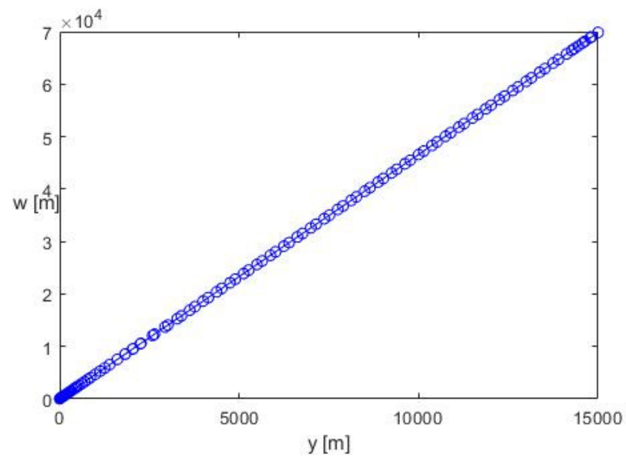

Figure 3: The variation of the plume width (w) in the y direction ($k = 50$).

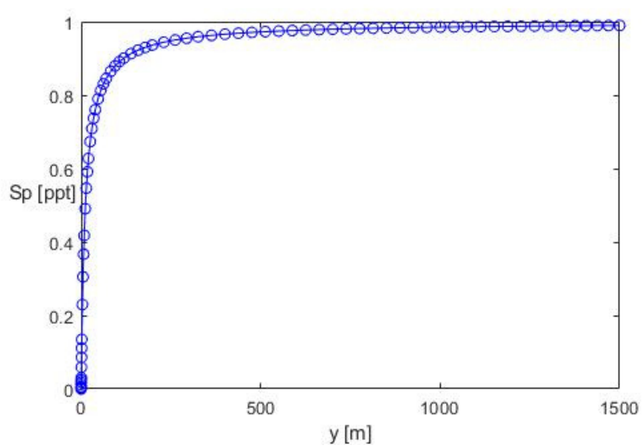

Figure 4: The variation of the salt concentration (S_p) in the y direction, in the plume ($k = 50$).

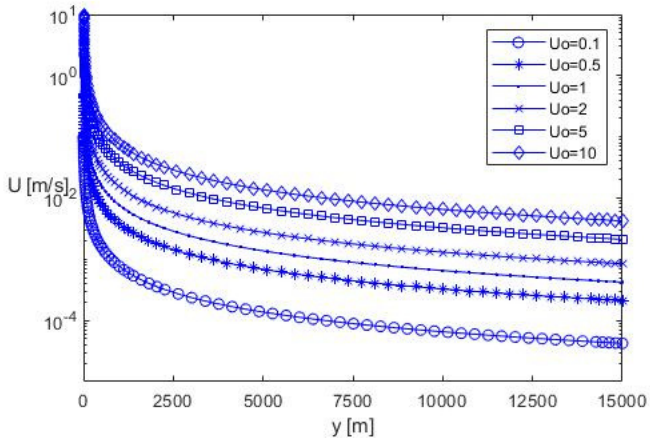

Figure 5: The variation of the average plume velocity (U) in the y direction for different values of U_0 ($k = 50$).

[2] J. Beckmann, M. Perrette, A. Ganopolski, *Simple models for the simulation of submarine melt for a Greenland glacial system model*, The Cryosphere, 12 (2018), 301 – 323.

[3] S.Y. Chao, W.C. Boicourt, *Onset of estuarine plumes*, J. Phys. Oceanogr., 16 (1986), 2137 – 2149.

[4] D.C. Chapman, S.J. Lenz, *Trapping of a coastal density front by the bottom boundary layer*, J. Phys. Oceanogr., 24 (1994), 1464 – 1479.

[5] P. Chatanantavet, M.P. Lamb, *Sediment transport and topographic evolution of a coupled river and river plume system: an experimental and numerical study*, J. Geophys. Res. Earth Surf., 119 (2014), 1263 – 1282.

[6] B. Deremble, *Convective plumes in rotating systems*, J. Fluid. Mech., 799 (2016), 27 – 55.

[7] M.J. Devlin, C. Petus, E. da Silva, D. Tracey, N.H. Wolff, J. Waterhouse, J. Brodie, *Water quality and river plume monitoring in the great barrier reef: an overview of methods based on ocean colour satellite data*, Remote Sens., 7 (2015), 12909 – 12941.

[8] A. Fabregat, W.K. Dewar, T.M. Ozgokmen, A.C. Poje, N. Wienders, *Numerical simulations of turbulent thermal, bubble and hybrid plumes*, Ocean Modelling, 90 (2015), 16 – 28.

[9] D.A. Fong, *Dynamics of freshwater plumes: observations and numerical modeling of the wind-forced response and alongshore freshwater transport*, Ph.D. Thesis, MIT (1998).

[10] D.A. Fong, W.R. Geyer, *Response of a river plume during upwelling favorable wind event*, J. Geophys. Res., 106 (2001), 1067 – 1084.

[11] D.A. Fong, M.T. Stacey, *Horizontal dispersion of a near-bed coastal plume*, J. Fluid. Mech., 489 (2003), 239 – 267.

[12] R.W. Garvine, *A dynamical system of classifying buoyant coastal discharges*, Continental Shelf Research, 15 (1995), 1585 – 1596.

[13] D.B. Haidvogel, A.R. Robinson, E.E. Schulman, *The accuracy, efficiency, and stability of three numerical models with application to open ocean problems*, J. Comput. Phys., 34 (1980), 1 – 53.

[14] D.B. Haidvogel, J.L. Wilkin, R. Young,*A semi-spectral primitive equation ocean circulation model using vertical sigma and orthogonal curvilinear horizontal coordinates*, J. Comput. Phys., 94 (1991), 151 – 185.

[15] A.R. Horner Devine, D.A. Fong, S.G. Monismith, T. Maxworthy, *Laboratory experiments simulating a coastal river inflow*, J. Fluid. Mech., 555 (2006), 203 – 232.

[16] A.R. Horner Devine, *The bulge circulation in the Columbia river plume*, Cont. Shelf Res., 29 (2009), 234 – 251.

[17] R.N. Miller, A.R. Robinson, D.B. Haidvogel,*A baroclinic quasigeostrophic open ocean model*, J. Comput. Phys., 50 (1983), 38 – 70.

[18] S. Nakada, S. Kobayashi, M. Hayashi, J. Ishizaka, S. Akiyama, M. Fuchi. M. Nakajima, *High-resolution surface salinity maps in coastal oceans based on geostationary ocean color images: quantitative analysis of river plume dynamics*, J. Oceanography, 74 (2018), 287 – 304.

[19] D. Nof, *On the dynamics of equatorial outflows with application to the amazon basin*, J. Marine Res., 39 (1981), 1 – 29.

[20] A.A. Osadchiev, P.O. Zavialov,*Lagrangian model of a surface-advected river plume*, Cont. Shelf Res., 58 (2013), 96 – 106.

[21] L.Y. Oey, G.L. Mellor,*Subtidal variability of estuarine outflow, plume, and coastal current: a model study*, J. Phys. Oceanogr., 23 (1993), 164 – 171.

[22] A. Poggioli, *Hydrodynamics and sediment transport at the river ocean interface: analytical and laboratory investigations*, Ph.D. Thesis, University of Washington, 2015.

[23] B.M. Satinsky, B.L. Zielinski, M. Doherty, C.B. Smith, S. Sharma, J.H. Paul, B.C. Crump, M.A. Moran, *The Amazon continuum dataset: quantitative metagenomic and metatranscriptomic inventories of the Amazon river plume*, June 2010, Microbiome, 2 (2014), 17.

[24] G. Shanmugam, *A global satellite survey of density plumes at river mouths and at other environments: plume configurations, external controls, and implications for deep-water sedimentation*, Petrol. Explor. Develop., 45 (2018), 640 – 661.

[25] A.E. Yankovsky, D.C. Chapman, *Generation of mesoscale flows over the shelf and slope by shelf wave scattering in the presence of a stable*, sheared mean current, J. Geophys. Res., 100 (1995), 6725 – 6742.

[26] A.E. Yankovsky, D.C. Chapman, *A simple theory for the fate of buoyant coastal discharges*, J. Phys. Oceanogr., 27 (1997), 1386 – 1401.

[27] A.E. Yankovsky, B.M. Hickey, A.K. Muenchow, *Impact of variable inflow on the dynamics of a coastal buoyant plume*, J. Geophys. Res., 106 (2001), 19809 – 19824.

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